DO NOW

Page 1

2.3 Evaluating Limits Analytically - Day 2

Strategies for Finding Limits:

Direct Substitution - For well-behaved functions that are continuous at c.

Simplify the rational expression and reduce (without factoring).

Simplify rational expressions by factoring and reducing to find a function that "agrees" with the given function at all points except where the original was undefined.

Rationalize the numerator of a fraction to reduce the fraction.

To use the special limits involving sine and cosine, the fraction may need to be manipulated.

Page 2

Examples:

1. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

$\frac{2^2+2-6}{2-2} = \frac{0}{0}$ indeterminate form

$\lim_{x\to 2} \frac{(x+3)\cancel{(x-2)}}{\cancel{x-2}}$

$\lim_{x\to 2} (x+3)$

$\boxed{5}$

2. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$\frac{\sqrt{4}-2}{4-4} = \frac{0}{0}$

$\lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{x-4(\sqrt{x}+2)}$

$\lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x}+2}$

$\frac{1}{\sqrt{4}+2}$

$\boxed{\frac{1}{4}}$

Page 3

3. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$\lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$

$\lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$

$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2}$

$\frac{1}{\sqrt{3+1}+2}$

$\boxed{\frac{1}{4}}$

4. $\lim_{\Delta x\to 0} \frac{3(x+\Delta x)-3x}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cancel{3x}+3\Delta x-\cancel{3x}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{3\cancel{\Delta x}}{\cancel{\Delta x}}$

$\lim_{\Delta x\to 0} 3$

$\boxed{3}$

Page 4

5. $\lim_{x\to 0} \frac{\tan x}{x}$

$\lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x}$

$\lim_{x\to 0} \frac{\sin x}{\cos x} \div x$

$\lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$1 \cdot \frac{1}{\cos 0}$

$1 \cdot \frac{1}{1}$

$\boxed{1}$

6. $\lim_{x\to 0} \frac{\sin 3x}{x}$

$\lim_{x\to 0} \frac{3}{3} \cdot \frac{\sin 3x}{x}$

$\lim_{x\to 0} 3 \boxed{\frac{\sin 3x}{3x}}$

$\boxed{3}$

Page 5

7. $\lim_{x\to 0} \frac{[1/(2+x)]-(1/2)}{x}$

$\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

$\lim_{x\to 0} \frac{\frac{2}{2(2+x)}-\frac{2+x}{2(2+x)}}{x}$

$\lim_{x\to 0} \frac{\frac{2-(2+x)}{2(2+x)}}{x}$

$\lim_{x\to 0} \frac{\cancel{2-2}-x}{4+2x} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{-x}{x(4+2x)}$

$\lim_{x\to 0} \frac{-1}{4+2x}$

$\frac{-1}{4+2(0)} = \boxed{\frac{-1}{4}}$

8. $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$

$\frac{2x}{5x}$

$\lim_{x\to 0} \frac{\sin 2x}{1} \cdot \frac{1}{\sin 5x}$

$\lim_{x\to 0} \frac{10x}{10x} \cdot \frac{\sin 2x}{1} \cdot \frac{1}{\sin 5x}$

$\lim_{x\to 0} \frac{2}{5} \cdot \frac{\sin 2x}{2x} \cdot \frac{5x}{\sin 5x}$

$\boxed{\frac{2}{5}}$

Page 6

HOMEWORK

pg 88; 45 - 63 odd, 69 - 81 odd, 89, 91